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Recent results on the fusion reactions between very heavy ions and nuclei †

Marc Lefort

Chimie Nucléaire, Institut Physique Nucléaire, 91406 Orsay, France

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Abstract. Results are given for measurements of complete fusion cross sections in ^{40}Ar and ^{84}Kr induced reactions on medium and heavy targets. After complete fusion, composite nuclei decay by particle and gamma ray emission and also, in large proportions, by binary fission. Cross sections were measured on the residual nuclei and on binary fissions following full momentum transfer, ie fissions issued from complete fusion. In the case of Ar ions, the fusion cross section is still a large part of the total reaction cross section and the restriction due to high angular momentum is not severe. It was found that the ratio $\sigma_{\mathrm{CF}}/\sigma_{\mathrm{r}}$ increases with the bombarding energy. If one assumes that σ_{CF} corresponds to the summation of partial waves until a critical value of the angular momentum l_{cr}, very large values of l_{cr} have been obtained (up to 140). With krypton projectiles there is also a large proportion of complete fusion if the compound nucleus is in the medium masses. On the other hand with targets of bismuth and uranium a very small cross section was observed for fission events following a fusion process. An intermediate situation was found in the case of holmium and tungsten targets. Most of the reaction cross section goes into incomplete fusion channels, with a large loss of kinetic energy. A discussion is given on the dynamical aspects of the collision between two heavy nuclei.

1. Introduction

1.1. Introductory comments on the fraction of the total reaction cross section going into complete fusion

It has been thought that in heavy ion induced reactions, there was a high probability for complete fusion of projectile and target, since the wavelength associated with high masses was rather large. Therefore, such reactions were considered as very useful in the synthesis of new isotopes and in the study of the statistical properties of highly excited nuclei. However, the experimental complete fusion cross sections which were measured with C, N, O and Ne projectiles (Kowalski *et al* 1968, Natowitz 1970a, b) did not coincide with the total reaction cross section and it was assumed that incomplete fusion processes occur with the projectiles of higher impact parameters.

Since $\sigma_{\mathrm{CF}}(E) < \sigma_{\mathrm{r}}(E) = \pi\lambda\!\!\!^{-2} \Sigma_{l=0}^{\infty} (2l+1)T_l$, one might consider that there is a stronger limitation in the summation than the limit given by assuming $T_l = 0$ at the largest l value for the partial wave.

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A simple assumption was the *sharp cut-off* model (Kowalski *et al* 1968) from which a limiting angular momentum J_{cr} may be extracted as a characteristic of complete fusion products:

$$\sigma_{\mathrm{CF}} = \pi \lambda\!\!\!^{-2} \sum_{l=0}^{l_{\mathrm{cr}}} (2l+1) T_l. \tag{1}$$

Ignoring the intrinsic spins of the projectile and target, and a distribution of orbital angular momenta:

$$P(L)\,\mathrm{d}L \begin{cases} = \dfrac{2L}{L_{\max}^2}\,\mathrm{d}L & \text{for } L < L_{\max} \\ = 0 & \text{for } L > L_{\max}, \end{cases}$$

the total reaction section is

$$\sigma_{\mathrm{r}} = \pi \lambda\!\!\!^{-2} (L_{\max}+1)^2. \tag{2}$$

The actual distribution of complete fusion products is assumed to have the same functional form but to be limited by J_{cr} rather than by $J_{\max}$. Thus:

$$\sigma_{\mathrm{CF}} = \pi \lambda\!\!\!^{-2} \sum_{l=0}^{L_{\mathrm{cr}}} (2l+1)$$

and therefore:

$$\sigma_{\mathrm{CF}} = \pi \lambda\!\!\!^{-2} (L_{\mathrm{cr}}+1)^2 \tag{3}$$

and the ratio $\sigma_{\mathrm{CF}}/\sigma_{\mathrm{r}}$ is given, after (2) and (3), according to:

$$\frac{\sigma_{\mathrm{CF}}}{\sigma_{\mathrm{r}}} = \left(\frac{L_{\mathrm{cr}}+1}{L_{\max}+1}\right)^2. \tag{4}$$

A more sophistical calculation for T_l values via the optical model would not change the basic concept expressed in (4) very much.

There has been much interest in this limiting value for orbital angular momenta (or in another form for limits in the impact parameters) since it controls the determination of complete fusion cross sections and it has great importance for the feasibility of reactions designed to produce new isotopes and particularly superheavy elements. When rather light projectiles were used for producing compound systems in a limited range of excitation energies, most of the measurements on cross sections seemed to behave in agreement with relation (4). The ratio $\sigma_{\mathrm{CF}}/\sigma_{\mathrm{r}}$ was observed to decrease as a function of increasing energy, as it was expected on the assumption that large l waves do not contribute any more to σ_{CF}. The data suggested that a roughly constant limiting angular momentum exists for survival of a complete fusion nucleus (Natowitz 1970a, b). This limit was estimated for medium masses around 35 to $40\hbar$, although rather large differences were obtained from one experiment to the other.

Nowadays there are new results available which lead to the conclusion that a sharp limit is not strictly correct and that it depends on the excitation energy for a given projectile–target combination (Natowitz *et al* 1972). Moreover, Zebelman and Miller (1973) on the one side, and Galin *et al* (1973), on the other side, have shown that sharp cut-off values of l_{cr} are different for different entrance channels. Therefore the processes that compete with complete fusion are not determined only by the equilibrium properties of the compound nuclei, but they seem *to entail the dynamics of the entrance channel.*

In table 1, we have tried to collect a large number of experimental data which show that l_{cr} is not at all a constant value and particularly that it can be much larger with very heavy projectiles.

Table 1. Compilation of measurements some complete fusion cross sections and derived l_{cr} values

Projectile	Target	Compound nucleus	E_i(lab) (MeV)	E^* (MeV)	l_{max}	l_{cr}	Reference
^{12}C	^{27}Al	^{39}K	44	41	22	20	Natowitz *et al* (1972)
^{12}C	^{27}Al	^{39}K	180	135	51	36	Natowitz *et al* (1972)
^{16}O	^{27}Al	^{43}Sc	105	80	40	35	Kowalski *et al* (1968)
^{20}Ne	^{27}Al	^{47}V	200	133	59	35	Pülhofer and Diamond (1972)
^{16}O	^{59}Co	^{75}Br	161	130	70	44	Kowalski *et al* (1968)
^{12}C	^{63}Cu	^{75}Br	126	110	52	27	Natowitz *et al* (1972)
^{16}O	^{63}Cu	^{79}Rb	168	138	71	40	Natowitz (1970a, b)
^{40}Ar	^{77}Se	^{117}Te	145	71	70	52	Galin *et al* (1973)
^{40}Ar	^{77}Se	^{117}Te	201	107	110	110	Galin *et al* (1973)
^{14}N	^{103}Rh	^{117}Te	121	107	70	52	Galin *et al* (1973)
^{14}N	^{103}Rh	^{117}Te	82	71	50	40	Galin *et al* (1973)
^{11}B	^{159}Tb	^{170}Y	115	107	62	39	Zebelman and Miller (1973)
^{12}C	^{158}Gd	^{170}Y	126	107	66	47	Zebelman and Miller (1973)
^{16}O	^{154}Sm	^{170}Y	137	107	72	58	Zebelman and Miller (1973)
^{40}Ar	^{121}Sb	^{161}Tm	200	86	93	81	Hanappe *et al* (1973)
^{40}Ar	^{121}Sb	^{161}Tm	300	164	150	130	Hanappe *et al* (1973)
^{84}Kr	^{74}Ge	^{158}Er	390	90	86	65	Gauvin *et al* (1973a, b)
^{40}Ar	^{118}Sn	^{158}Er	200	90	92	80	Gauvin *et al* (1973a, b)
^{40}Ar	^{165}Ho	^{205}At	226	97	110	102	Hanappe *et al* (1973)
^{40}Ar	^{165}Ho	^{205}At	300	160	166	139	Hanappe *et al* (1973)
^{12}C	^{197}Au	^{209}At	86	63	50	42	Bimbot *et al* (1968)
^{12}C	^{197}Au	^{209}At	126	101	64	51	Natowitz (1970a, b)
^{16}O	^{197}Au	^{213}Fr	168	122	90	74	Natowitz (1970a, b)
^{20}Ne	^{209}Bi	^{229}Np	210	135	104	79	Natowitz (1970a, b)
^{40}Ar	^{209}Bi	^{249}Md	250	80	122	110	Hanappe *et al* (1973)
^{40}Ar	^{238}U	^{278}110	250	82	117	92	Hanappe *et al* (1973)
^{40}Ar	^{238}U	^{278}110	300	125	166	128	Hanappe *et al* (1973)
^{40}Ar	^{238}U	^{278}110	400	204	240	160	Sikkeland (1968)
^{84}Kr	^{165}Ho	^{249}Lw	450	62	139	~ 70	Lefort *et al* (1973a, b)
^{84}Kr	^{186}W	^{270}110	502	105	173	~ 67	Lefort *et al* (1973a, b)
^{84}Kr	^{209}Bi	^{293}119	502	52	157	<20	Lefort *et al* (1973a, b)

1.2. Theoretical limitations for complete fusion

Let us first define what is called *complete fusion*. It corresponds to interactions where all the nucleons from both partners are joined together for a time that is much longer than the collision time. Then after some delay, an intermediate (composite nucleus) decays into the final products. It does not necessarily mean a complete equilibrium of states in a definite potential well. There are a number of approaches for explaining that complete fusion cross sections might be significantly less than total reaction cross sections.

1.2.1. Yrast levels. At a particular excitation energy obtained in the product nucleus, there can be no levels with angular momentum higher than the limiting value (Grover

1967). Other collisions which would have produced nuclei with higher angular momentum states *must* therefore proceed through other channels. The limitation appears in the compound states whatever would be the entrance channel, as far as the angular momentum population is the same. There have been several suggestions for calculating the energies of the Yrast levels.

(i) If it is assumed that the entire excitation energy is manifest as rotational energy of a spherical nucleus with a rigid body inertia, $\mathscr{I}_{\mathrm{rig}}$ (rigid rotor), then J_{cr} is obtained directly (Grover 1967) from $E^* = E_{\mathrm{rot}} = J_{\mathrm{cr}}^2/2\mathscr{I}_{\mathrm{rig}}$. ($L_{\mathrm{cr}}$ and J_{cr} are the same if one neglects spins of target and projectiles.)

(ii) A more realistic calculation was based on the single particle model by Hillmann and Grover (1969).

(iii) Assuming a macroscopic model, Cohen *et al* (1963) have described the nuclear shape as a liquid drop which becomes increasingly non-spherical as J increases and they have deduced J_{cr} from the rotational energy of this system having a larger moment of inertia than the rigid sphere. The results for rare earth compound nuclei seem to agree with such a theoretical limit, as shown on figure 2 of the paper by Lefort *et al* (1963).

1.2.2. Arbitrary shapes for the fusing system. Complete fusion leads more readily to an ellipsoidal-shaped nucleus. The derivation of a limiting angular momentum for the survival was made by Kalinkin and Petkov (1964) on the assumption that, for the corresponding limiting eccentricity ϵ, the surface tension would be too small to counteract Coulomb energies, and the system could not be stable any longer. The eccentricity $\epsilon = (1-b^2/a^2)^{1/2}$ is related directly to the distance of closest approach $R_1(l, E)$ and is a function of orbital angular momentum and projectile energy. A detailed survey of this model shows that, for a given nucleus, J_{Cr} decreases slowly when the excitation energy increases. There are at the present time a number of experimental results that disagree with this conclusion.

1.2.3. Rotating liquid drop. The third approach assumes that for all impact parameters which contribute to the total cross section, a compound nucleus is formed and that fission may compete with particle emission in the de-excitation process. Since the fission barrier is a function of angular momentum, it is suggested (Blann and Plasil 1972) that the non-compound portion of total reaction cross sections is a type of fission. More specifically, the complete fusion cross section is expressed as

$$\sigma_{\mathrm{CF}} = \pi\lambda^2 \sum_{l=0}^{\infty} (2l+1)T_l(\mathrm{CF})$$

where the quantities $T_l(\mathrm{CF})$ are given by $T_l P_l(\mathrm{CF})$ where T_l is the usual transmission coefficient for partial wave l and $P_l(\mathrm{CF})$ the probability that the compound nucleus formed from the lth partial wave *survives* de-excitation without fissioning. $P_l(\mathrm{CF})$ depends only on the compound nucleus that is formed and should be independent of *entrance channel.* In other words $\sigma_{\mathrm{CF}}/\pi\lambda^2$ should be the same and equal to

$$\sum (2l+1)T_l P_l(\mathrm{CF}),$$

for any entrance channel if the transmission coefficients are the same. With this concept in mind, values of $E(J)$, the minimum energy of a rotating drop with angular momentum J at equilibrium deformation, may be obtained (liquid drop Yrast energies), and also $E_{\mathrm{sp}}(J)$, the energy of the saddle-point shape. The fission barrier of the rotating drop $B_f(J)$ is the difference $E_{\mathrm{sp}}(J) - E(J)$. Since $E_{\mathrm{sp}}(J)$ increases as a function of J at a lower rate than $E(J)$ because of the much more deformed shape of the saddle-point, $B_f(J)$ decreases when J increases, and a limit $B_f(J)_{\mathrm{cr}} = 0$ can be attained. As has been pointed out by Blann and Plasil (1972), the assumption of compound nucleus formation may not be valid as B_f tends to zero, but calculated $\sigma_{\mathrm{CF}}/\sigma_{\mathrm{r}}$ results remain unaffected.

When J_{cr} is defined by $B_f(J) = 0$, its value is strongly dependent on the fissibility parameter x and the maximum value is found at $95\hbar$ for $x = 0{\cdot}5$, ie for medium A nuclei (Plasil 1972).

In a classical approximation, one might express σ_{CF} by the relation:

$$\sigma_{\mathrm{CF}} = \pi R_{\mathrm{cr}}^2(1-E_{\mathrm{Io}}/E),$$

where E_{Io} is the interaction barrier, taking for R_{cr} the relative separation corresponding to J_{cr}, as given by the angular momentum definition: $J_{\mathrm{cr}} = R_{\mathrm{cr}}[2\mu(E-E_{\mathrm{Io}})]^{1/2}$, then

$$\sigma_{\mathrm{CF}} = \pi \frac{J_{\mathrm{cr}}^2}{2\mu E}. \tag{5}$$

On the excitation function, $\sigma_{\mathrm{CF}}(E)$ (figure 1), loci of fixed angular momentum J_{cr} lie on a hyperbola for energies higher than the value corresponding to $J_{\mathrm{max}} = J_{\mathrm{cr}}$. Such a hyperbola divides the plane in two regions: to the right the system has too much angular momentum, and collsions would lead to a *non-fusion* process; to the left there exists a fission barrier, but the compound nucleus still has a large probability for fissioning in the region between $B_f(J) = 0$ and $B_f(J) = S_{\mathrm{n}}$, where S_{n} is the separation energy of the less-bounded particle.

Now, we are going to present in the framework of this model, some experimental data obtained with Ar and Kr projectiles, which bring in large orbital angular momenta. They show that none of the previous explanations is sufficient in itself to explain the behaviour of complete fusion cross sections as a function of energy.

2. Experimental data

2.1. Experimental measurements of σ_{CF} on the system ($^{40}Ar+{}^{121}Sb$)

When bombarding ^{121}Sb by ^{40}Ar projectiles, the compound system is ^{161}Tm. Cross sections have been measured (Gauvin *et al* 1973a, b) at various bombarding energies for the residual nuclei resulting from x neutron emissions (Ar, xn), from one proton and y neutron emission (Ar, pyn), from two protons (or one α particle), and z (or $z-2$) neutrons emission (Ar, 2p, zn) (Ar, $\alpha(z-2)$n) and from (Ar, 3p, wn) reactions. The results were obtained for ^{118}Sn and ^{121}Sb targets.

After a careful study (Gauvin *et al* 1973a, b) of the reaction products decaying from compound nuclei in this region of the rare earths, it was concluded that the summation for all the preceding reactions yields the complete fusion cross section, except for the fraction which decays through fission channels. We note them as

$$\sigma_{\mathrm{cf}} = \sum (\mathrm{Ar}, x\mathrm{n}) + (\mathrm{Ar}, \mathrm{p}y\mathrm{n}) + \dots.$$

However, although the fission cross section has been found (Sikkeland *et al* 1962) at a rather low rate of a few millibarns in the case of ^{12}C or ^{16}O induced reactions in this region of the rare earths, it was suspected that it might be much higher with much heavier projectiles like ^{40}Ar. Therefore, fission cross sections were measured on a natural Sb target bombarded by ^{40}Ar at different energies. Fission fragments were

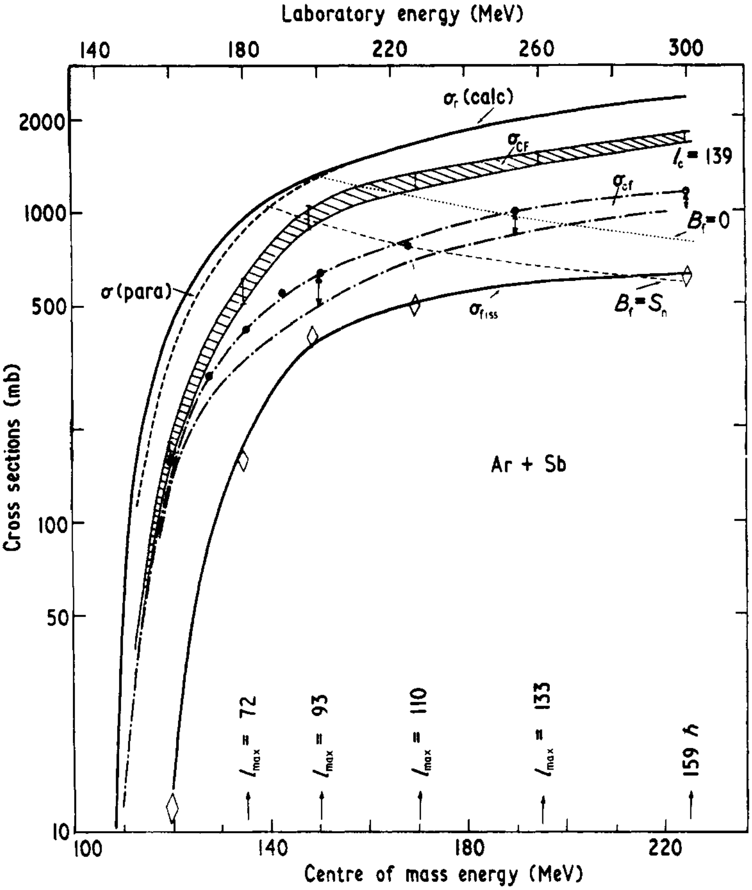

Figure 1. Complete fusion cross sections in the reaction (Ar + Sb) as a function of energy. Lower scale: centre of mass energy and angular momenta. Upper scale: laboratory energy. σ_r and σ_r(para) are calculated total reaction cross sections. σ_{fiss} is drawn between the experimental points for the fission cross sections. σ_{cf} has been obtained by summing cross sections for all residual nuclei. Possible errors are shown by arrows and the lower limit is the broken curve. The hatched area, denoted σ_{CF}, shows where the complete fusion cross section is located. Calculated curves $B_f = 0$ and $B_f = S_n$ are explained in the text.

counted as coincidences by detectors located at correlation angles corresponding to a full momentum transfer to a complete fusion mass, followed by a prompt fission. A detailed account of these results is given elsewhere (Lefort *et al* 1973). Fission cross sections from the compound nucleus have been measured at centre of mass energies 120, 134, 148, 168 and 226 MeV, and a curve has been drawn for $\sigma_{\text{fiss}}(E)$. The results are given on figure 1. In principle the sum $(\sigma_{\text{cf}} + \sigma_{\text{fiss}})$ should represent the total complete fusion cross section σ_{CF}, since all the fission events correspond to a full momentum transfer and behave like fission fragments from $^{161}\text{Tm}^*$. The curve $\sigma_{\text{CF}}(E)$ is also given on figure 1. Now a number of remarks can be made on these results, presented under the critical angular momentum concept.

(i) Although the complete fusion cross section is never as large as the total reaction cross section, the ratio $\sigma_{\text{CF}}/\sigma_r$ stays more or less constant as a function of energy. Then, according to relation (4), L_{cr} should increase continuously as a function of energy, since $L_{\max}$ is directly proportional to $(E-E_{\mathrm{Io}})^{1/2}$. This is clearly shown in table 2 where the limiting value is calculated according to (4), either with $\sigma_{\mathrm{cf}}(l_{\mathrm{cf}})$ or with

$$\sigma_{\mathrm{CF}} = \sigma_{\mathrm{cf}}(l_{\mathrm{cf}})+\sigma_{\mathrm{fiss}}(l_{\mathrm{cr}}).$$

Very high l_{cr} values are found for the highest energy (300 MeV in the laboratory), since for $l_{\max} = 159$, $l_{\mathrm{cr}} = 130$. Most of the values are much larger than $50\hbar$ which has sometimes been considered as a limit in the rare earth region.

Table 2. Complete fusion cross sections and limits for the angular momentum $J_{\mathrm{cr}} = l_{\mathrm{cr}}\hbar$. in the case of the reaction (^{40}Ar + Sb) (see text for l_{cf})

Centre of mass energy (MeV)	E^* (MeV)	(Ar, xn, yp . . .) (mb) lower limit–higher limit	σ_{CF}(mb)	$l_{\max}$	l_{cf}	$\langle l\rangle_{\mathrm{cr}}$
120	58	125–150	150–165	49	28	29
134	72	380–420	500–560	72	49	57
148	86	500–580	900–1000	93	61	81
168	106	760–850	1150–1250	110	79	90
195	133	1000–1100	1500–1600	133	97	117
226	164	1150–1250	1700–1800	159	112	130

(ii) The cross section for incomplete fusion can be estimated as $\sigma_{\mathrm{r}}-\sigma_{\mathrm{CF}}$ and it seems to reach a steady value around 550 mb. This is in contradiction with relation (5) made on the basis of J_{cr} calculated after $B_{\mathrm{f}}(J) = 0$. Limits for σ_{CF} and σ_{cf} on figure 3 should roughly follow the curves noted $B_{\mathrm{f}} = 0$ and $B_{\mathrm{f}} = S_{\mathrm{n}}$.

(iii) The fission cross section threshold seems to be higher than the complete fusion threshold, probably because in the vicinity of the interaction barrier, angular momenta are not large enough to diminish the fission barrier to such an extent that $\Gamma_{\mathrm{f}}/\Gamma_{\mathrm{n}}$ becomes important. Such an effect is not observed for heavier compound systems where $B_{\mathrm{f}}(J = 0)$ is already relatively low.

2.2. *Further data on complete fusion for Ar induced reactions*

Besides the results reported in § 2.1, there are other measurements on fission cross sections which show that the concept of an angular momentum limit should be considered with caution. Some years ago, Sikkeland (1967, 1968) obtained $\sigma_{\mathrm{CF}}/\sigma_{\mathrm{r}}$ by fragment–fragment angular correlation measurements in the case of (^{40}Ar + ^{238}U), where σ_{r} was measured as the total fission cross section and σ_{CF} as that fraction of the fission cross section which corresponds to full momentum transfer. At a laboratory energy of 400 MeV, σ_{CF} was around $\frac{1}{2}\sigma_{\mathrm{r}}$, which in terms of J_{cr}, gives a limiting value of the order of 160 $\hbar$.

The same technique has been applied by Hanappe *et al* (1973) in the system (^{40}Ar + ^{165}Ho) for which incomplete fusion has a very small probability of leading to fission events. Therefore σ_{fiss} is entirely the result of the fission de-excitation from a complete fusion nucleus. In addition, we have measured (Le Beyec *et al* 1971) the cross section for (Ar, xn) reactions in a similar case (^{40}Ar + ^{164}Dy), and it is certainly lower than

100 mb. σ_{CF}/σ_r and J_{cr} values are given for two energies in table 3. Once again, l_{cr} reaches 139 $\hbar$ at the highest excitation energy. In addition results from Hanappe *et al* (1973) are given for Ar in induced reactions on Mo, Sb, Bi and U targets. Some are not very accurate and the limiting value of l is shown in brackets.

Table 3. Fission cross sections from compound systems formed by Ar induced reactions, after Hanappe *et al* (1973)

Target	Compound	Laboratory energy (MeV)	Centre of mass energy (MeV)	Interaction barrier	E^* (MeV)	B_f (MeV)	σ_F(mb)	σ_r	σ_F/σ_r	l_{max}	l_{cr}
^{nat}Mo	Nd	200	140	93·5	98	39	145±30	1400	0·1	96	(82)
^{nat}Mo	Nd	300	209·5	93·5	169	39	163±15	2350	0·07	151	(83)
^{nat}Sb	Tm	162	122	109	58	28·5	103±1	450	0·02	53	(<53)
^{nat}Sb	Tm	179	135	109	72	28·5	187±20	840	0·22	74	(60)
		199	150	109	86	28·5	510±80	1250	0·4	94	(80)
		226	170	109	106	28·5	535±50	1620	0·32	114	(90)
		300	226	109	164	28·5	620±60	2370	0·26	158	(130)
^{165}Ho	^{205}At	226	182	135	97	13	860±90	1348	0·64	110	102
^{165}Ho	^{205}At	300	242	135	160	13	1430±140	2300	0·62	166	139
^{209}Bi	^{249}Md	250	210	158	80	~0	1110±200	1380	0·81	122	110
^{238}U	$^{278}110$	250	214	171	82	<0	766±150	1230	0·62	117	92
^{238}U	$^{278}110$	300	257	171	125	<0	1220±120	2043	0·60	166	128

A very different method has been used recently by Galin *et al* (1973) in their study of the de-excitation of the compound nuclei ^{117}Te by charged particle emission. Such a compound system has been formed by two sets of reactions ($^{40}Ar+{}^{77}Se$) and ($^{14}N+{}^{103}Rh$) at two different excitation energies. By a very clever analysis of the angular distribution of energetic α particles, the authors have shown that a limit in J should be considered, but its value depends strongly on the entrance channel (for the same excitation energy) and on the excitation energy for the same bombarding ion, as shown in tables 1 and 5. For (Ar + ^{77}Se) at the highest energy, J_{cr} is as large as 70 $\hbar$, in agreement with the results obtained in the same range of excitation energies for ($^{40}Ar+{}^{121}Sb$).

2.3. Remark on the significance of fission cross section measurements from full momentum transfer technique

The observation of two fission fragments at correlation angles corresponding to a full momentum transfer might not be considered as a proof in favour of the formation of a long-lived compound nucleus. One may think, for example, that a substantial transfer of momentum occurs, due to dynamical effects during the violent interaction between the heavy projectile and the target. The system may disintegrate shortly into fission fragments without pairing through the stage of a definite nucleus. However, extensive results by Hanappe *et al* (1973) show that in the case of Ar ions, energy distributions of fission fragments behave just like those resulting from lighter projectiles (symmetry, most probable kinetic energy around 200 MeV etc), for which fission channels are known as decay processes from a compound nucleus. Therefore, it seems reasonable to admit that when a full momentum transfer is deduced from the kinematics study, a composite system has been formed and has lasted for some time, even if full thermodynamical equilibrium has not been entirely reached. We shall see that this is no longer the case for some krypton induced reactions.

3. Krypton ion induced reactions: complete fusion and fission of the compound system

It is difficult, at present, to obtain a large set of data in order to clarify what fraction of the reaction cross section goes into complete fusion, when Kr projectiles are used. The lightest compound system which can be formed is already a rare earth and we know that fission occurs with a large yield. But it is not easy to distinguish fission fragments from elastic and inelastic events since masses are very similar. Then σ_{fiss} is difficult to measure. With heavier targets, the fission cross section represents the total reaction cross section, but the mass distribution and the energy distribution of the fragments is so wide that it becomes difficult to distinguish between a fission event from a complete fusion system and a fission subsequent to an incomplete fusion.

3.1. *Light targets*

The reaction (^{84}Kr + ^{74}Ge) has been studied (Gauvin *et al* 1973b) by the measurement of the excitation function for the cross section of erbium residual nuclei resulting from (^{84}Kr, xn) reactions. Precise comparison has been made between the excitation functions of the reactions ^{118}Sn(^{40}Ar, 5n)^{153}Er and ^{74}Ge(^{84}Kr, 5n)^{153}Er. For both cases the angular momentum populations are very similar. At the peak of the excitation functions $\sigma(\text{Kr, 5n})/\sigma_{\text{r}} = 0{\cdot}08 \pm 0{\cdot}02$ and $\sigma(\text{Ar, 5n})/\sigma_{\text{r}} = 0{\cdot}11 \pm 0{\cdot}02$. Assuming that the other de-excitation channels behave in a similar manner, one can estimate that $(\sigma_{\text{CF}}/\sigma_{\text{r}})$ for the case of Kr projectiles is roughly 30% lower than for the case of Ar. It would correspond to 0·5 at a centre of mass energy of 182 MeV. According to relation (4) a critical value of 65 $\hbar$ could be deduced at an excitation energy around 90 MeV, where $J_{\text{cr}} = 81\ \hbar$ for Ar induced reactions. The qualitative conclusion which can be drawn is that large values of J still seem possible for the compound nucleus.

3.2. *Heavy targets*

Measurements of fission cross sections have been made with a number of heavy targets, from ^{165}Ho up to ^{238}U. Unambiguous results (Lefort *et al* 1973b) were obtained only for the cases of ^{209}Bi and ^{238}U, where fission fragments were detected as coincidences. The detectors were located in order to collect the fission fragments emitted by a complete fusion system ($A = 294$ in the case of Bi; $A = 322$ in the case of U), taking account of full momentum transfer and assuming binary fission with characteristics in the range of those predicted by Nix (1969). Therefore it was believed that σ_{fiss} would give a measurement of σ_{CF}. Since the technique is the same as that which was successful in the case of (^{40}Ar + ^{238}U), there is no doubt that a clear answer could be given. At a beam energy of 500 MeV, results are given in table 4, as well as some of the data obtained by the same technique with Ar ions. More preliminary results are also shown for the cases of (Kr + W) and (Kr + Ho), which are interesting since the expected composite systems are very close to systems obtained respectively by Ar on U and by Ar on Bi.

Typically, figure 2 illustrates the difference between the results obtained on the systems (Ar + U) and (Kr + W), where the composite system has $Z = 110$ in both cases.

Table 4. Fusion–fission cross sections σ_f

Pairs	Compound nucleus	Centre of mass barrier (MeV)	Centre of mass energy (MeV)	Expected correlation θ_x θ_y	Measured correlation θ_x θ_y	σ_f (mb)	σ_r (mb)
Kr + ^{165}Ho	^{249}103	263 ± 6	305 ± 3	50–(50–37)	50–(75–35)	~200	800
Kr + Bi	^{293}119	312 ± 7	357 ± 4	54–(54 – 42)	54–(86–24)	<40	800
Kr + U	^{322}128	335 ± 7	370 ± 4	57–(57–45)	57–(87–40)	<10	610
Kr + ^{186}W	^{270}110	284 ± 6	346 ± 4	54–(52–40)	54–(80–25)	~150	990
Ar + U	^{278}110	110 ± 5	254 ± 3	65–(67–54)	60–(75–50)	1220	2040

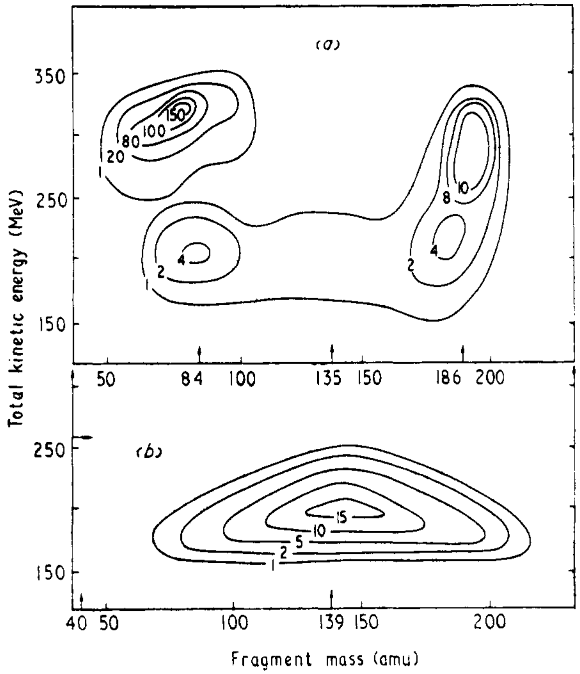

Figure 2. Total kinetic energy against fragment masses in fission induced reactions. Contour lines indicate the number of events. Arrows show masses of projectile and target and of binary symmetric fission fragment. Upper part: (Kr + ^{186}W). Elastic and inelastic scattering are shown. Lower part: (Ar + U).

The experiments were made at roughly the same excitation energy and the same angular momentum distribution. However, in the case of krypton ions, the bombarding energy is closer to the Coulomb barrier. A large number of fission fragments issued from a binary fission were observed around mass $A = 139$ and total kinetic energy peaking at 200 MeV in the case of argon induced reactions. On the contrary, very few events could be found in the same area, in the case of krypton projectiles, and most of the reaction products appeared around mass 84 and mass 186, as if they were the result of a very inelastic process. The same comparison was made between (Kr + ^{165}Ho) and (Ar + ^{209}Bi) at a comparable excitation energy (90 MeV).

The fact that σ_f/σ_r is so small and furthermore that no binary fission could be identified in the case of uranium, is particularly troublesome. In terms of l_{cr}, very low limits were deduced as shown in table 1.

Sierk and Nix (1973) have recently made an attempt to explain these results by describing the dynamics of the fusion process. Even without including any viscosity effect, they calculated, as a function of a fissility parameter, the amount of incident energy necessary for symmetric systems to *fuse* in a dynamic way to a configuration more compact than the liquid drop model saddle-point shape. For values of x less than 0·72, no energy greater than the interaction barrier is needed and indeed we have measured reasonable complete fusion cross sections for the formation of erbium nuclei by bombarding ^{74}Ge with ^{84}Kr. But above $x = 0{\cdot}72$, the energy rises steeply and in the case of two colliding ^{150}Nd nuclei, the result shows that more than 100 MeV of energy greater than the interaction barrier is needed to drive the system in a short time to a nearly spherical shape. Such a shape is required to allow significant exchange of nucleons between the interacting nuclei.

Another approach has been used by Wilczynski (1973), on the basis that complete fusion occurs only when attractive nuclear forces are larger than the repulsive Coulomb and centrifugal forces. Nuclear forces acting at the touching point were described by the derivative of the surface energy of two liquid drops. Again, the author found that very little cross section would go into fusion in the case of (Kr + U).

The concept that dynamical force equilibrium in the entrance channel is the determining criterion for the critical angular momentum has been in our mind for several years and the conclusion is a brief schematic account of this idea.

4. Conclusion: importance of the entrance channel for the complete fusion between two complex nuclei

The results which have been obtained recently with very heavy ions like Ar and Kr show that it is doubtless true that there is some critical range of $l\hbar$ above which the complete fusion products cannot stick together. The Yrast line may be reached in some cases. The rotating nuclear fission barrier $B_f(J)$ might also approach zero. However, J_{cr} *increases regularly when the excitation energy is increased* and this is not predicted by the rotating liquid drop model. In the Blann and Plasil model, J_{cr} should increase with projectile energy until J reaches the value where Γ_f becomes important and for much higher energies it should decrease. Moreover, there are a number of results which show that, with the same excitation energy for the same compound nucleus, the complete fusion cross sections are different. Such an influence due to dynamical processes in the entrance channel has been demonstrated by Zebelman and Miller (1973) with ^{11}B, ^{12}C and ^{16}O and is even more dramatic when Ar ions are the projectiles. In order to illustrate this point, table 5 gives the results from these authors as well as new data at the same excitation energy.

It seems to us that a more realistic approach to the problem of complete fusion and incomplete fusion would be to describe correctly the potential energy of the two colliding nuclei for each l wave (Basile *et al* 1972). In the overlapping tails of nuclear matter density, interactions between nucleons of both nuclei might occur. An important question is: is there a dip or not in the interaction potential for a time that is long enough to allow *non-adiabatic* exchanges and then a single compound nucleus formation?

Table 5. Influence of the entrance channel on complete fusion fraction of the reaction cross section

Reaction	Compound nucleus	Excitation energy (MeV)	l_{max}	l_{cr}
$^{11}B+{}^{159}Tb$			62	39
$^{12}C+{}^{158}Gd$	$^{170}_{70}Yb$	107	66	47
$^{16}O+{}^{154}Sm$			72	58
$^{40}Ar+{}^{121}Sb$	$^{161}_{69}Tm$	106	110	90
$^{84}Kr+{}^{74}Ge$	$^{158}_{68}Er$	100	86	~65

This depends strongly on the centrifugal potential and on the balance between Coulomb and nuclear forces.

Very recently, Galin *et al* (1973, unpublished) have tried to apply this concept by using Bruckner's nuclear potentials for the two colliding nuclei. The potential energy of the system is calculated in the framework of the sudden approximation, as a function of the distance between centres of the two approaching nuclei. If one adds the Coulomb potential, one obtains a curve representing the potential energy as a function of distance between centres for *s* waves. Then, including the centrifugal potential, curves are drawn for each *l* wave.

A systematic study made for a number of nuclei and projectiles and a comparison with experimental measurements of l_{cr} values has shown that the limitation to complete fusion seems very strongly related *to a critical distance between the two centres* and not to a single critical value of angular momentum. Such a critical distance R_{cr} is unique

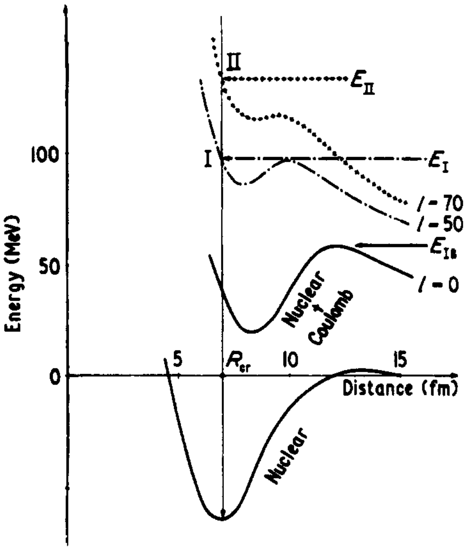

Figure 3. Schematic example of potential energy curves for two colliding nuclei. R_{cr} is defined in the text. E_{IB} is the interaction barrier corresponding to the energy necessary to overcome the potential barrier for $l = 0$. E_I is the kinetic energy in the centre of mass system when the corresponding l_{cr} is found at point I, $l_{cr} = 50$. E_{II} is the kinetic energy in the centre of mass system when the corresponding l_{cr} is found at point II, $l_{cr} = 70$.

for a given pair of colliding nuclei, and might be chosen as the distance corresponding to the bottom of the nuclear potential as shown on figure 3. At a given bombarding energy (centre of mass) there is a particular l wave curve which reaches on the energy ordinate that bombarding energy for the distance abscissa equal to R_{cr}. This curve defines the critical angular momentum. For a smaller l, the bombarding energy intersects the curve at a smaller distance than R_{cr} and a compound nucleus is formed. For a larger l, the intersection occurs at a larger distance and there is not enough nuclear interaction so that the Coulomb repulsive potential dominates and most of the incoming nucleons are repelled. R_{cr} changes of course from one projectile to another, and, for a given pair of partners, l_{cr} increases with the bombarding energy, ie with l_{max} or with excitation energy, as has been observed.

If such a picture is true, the Coulomb repulsive potential is so high for very heavy ions that the overall potential dip tends to vanish. Then, even for low l values, the distance of approach at which the potential against distance curves encounter the kinetic energy ordinate is always larger than R_{cr} and complete fusion should be strongly inhibited. It is certainly necessary to built accurate curves for the nuclear potential in order to obtain exact values of the distance for the minimum. Such work is now in progress by Galin *et al* for all cases where experimental results are available. Such an approach has the advantage of taking account of the entrance channels, of the excitation energy and, with some improvements, of the possible deformation of colliding nuclei.

References

Basile R, Galin J, Guerreau D, Lefort M and Tarrago X 1972 *J. Phys., Paris* **33** 9

Bimbot R, Lefort M and Simon A 1968 *J. Phys., Paris* **29** 563

Blann M and Plasil F 1972 *Phys. Rev. Lett.* **29** 303

Cohen S, Plasil F and Swiatecki W J 1963 *Proc. 3rd Conf. on Reactions Between Complex Nuclei* (Berkeley: University of California Press) pp 325–30

Galin J, Gatty B, Guerreau D, Rousset C, Schlotthauer-Voos U and Tarrago X 1973 *Institut Physique Nucléaire, Orsay Reports* IPNO-RC-73-03, IPNO-RC-73-07

Gauvin H, Le Beyec Y and Porile N T 1973a *Phys. Rev.* in the press

Gauvin H, Hahn R L, Le Beyec Y and Lefort M 1973b in preparation

Grover J R 1967 *Phys. Rev.* **157** 832

Hanappe F, Ngô C, Péter J and Tamain B 1973 *3rd Symp. on the Physics and Chemistry of Fission, Rochester* IAEA/SM-174/42

Hillman M and Grover J R 1969 *Phys. Rev.* **183** 1303

Kalinkin B N and Petkov I Z 1964 *Acta Phys. Polon.* **25** 265

Kowalski L, Jodogne J and Miller J M 1968 *Phys. Rev.* **169** 894

Le Beyec Y, Lefort M and Vigny A 1971 *Phys. Rev.* C **3** 1268

Lefort M, Le Beyec Y and Péter J 1973a *Institut Physique Nucléaire, Orsay Report* IPNO-RC-73-04

Lefort M, Ngô C, Péter J and Tamain B 1973b *Nucl. Phys.* in the press

Natowitz J B 1970a *Phys. Rev.* C **1** 623

—— 1970b *Phys. Rev.* C **1** 2157

Natowitz J B, Chulick E T, Namboodiri M N 1972 *Phys. Rev.* C **6** 2133

Nix J R 1969 *Nucl. Phys.* A **130** 241

Plasil F 1972 *Proc. Eur. Conf. on Nuclear Physics, Aix-en-Provence* (*J. Phys. Paris Suppl.* 2 51)

Pülhofer F and Diamond R M 1972 *Nucl. Phys.* A **191** 561

Sierk A J and Nix J R 1973 *3rd Symp. on the Physics and Chemistry of Fission, Rochester* IAEA-SM-174/74

Sikkeland T 1967 *Ark. Fys.* **36** 539

—— 1968 *Phys. Lett.* **27B** 277

Sikkeland T, Viola V E and Haines E L 1962 *Phys. Rev.* **125** 1350

Wilczynski J 1973 *Nucl. Phys.* in the press

Zebelman A M and Miller J M 1973 *Phys. Rev. Lett.* **30** 27